Unconventional Hall effect near charge neutrality point in a two-dimensional electron-hole system
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O. E. Raichev,1 G. M. Gusev,2 E. B. Olshanetsky,3 Z. D. Kvon,3 N. N. Mikhailov,3 S. A. Dvoretsky,3 and J. C. Portal4,5,6

1Institute of Semiconductor Physics, NAS of Ukraine, Prospekt Nauki 41, 03028 Kiev, Ukraine
2Instituto de Física da Universidade de São Paulo, 13596-170 São Paulo, SP, Brazil
3Institute of Semiconductor Physics, Novosibirsk 630090, Russia
4LNCMI-CNRS, UPR 3228, BP 166, 38042 Grenoble Cedex 9, France
5INSA Toulouse, 31077 Toulouse Cedex 4, France
6Institut Universitaire de France, 75005 Paris, France

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The transport properties of the two-dimensional system in HgTe-based quantum wells containing simultaneously electrons and holes of low densities are examined. The Hall resistance, as a function of perpendicular magnetic field, reveals an unconventional behavior, different from the classical N-shaped dependence typical for bipolar systems with electron-hole asymmetry. The quantum features of magnetotransport are explained by means of numerical calculation of the Landau level spectrum based on the Kane Hamiltonian. The origin of the quantum Hall plateau \( \sigma_{xy} = 0 \) near the charge neutrality point is attributed to special features of Landau quantization in our system.

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I. INTRODUCTION

The renewed interest to the study of the integer quantum Hall effect (QHE) has been manifested recently in investigation of the anomalous QHE state in graphene, which provides electron or hole excitations near the Dirac point of a double-cone energy spectrum.1,2 It has been predicted and observed that quantized values of the Hall conductance \( \sigma_{xy} = ve^2/h \) correspond to filling factors \( \nu = 4(n + 1/2) \), where \( n = 0, \pm 1, \pm 2, \ldots \) are integers and the factor of 4 accounts for double degeneracy in both spin and valley numbers. The half-integer form reflects a specific property of Landau quantization for massless Dirac fermions. In particular, zero Landau level, whose position coincides with the Dirac point, is composed from half hole and half electron states, so the Hall conductance indicates a smooth transition from holelike (positive \( \nu \)) to electronlike (negative \( \nu \)) behavior as the Fermi energy goes up through this point. In very strong magnetic fields the spin degeneracy (and, possibly, the valley degeneracy) is lifted, which means opening the gap at the Dirac point. Theoretical models considering electron transport in these conditions can be classified in two groups: quantum Hall metal (spin-first lifting scenario) and quantum Hall insulator (valley-first lifting scenario).3 In the spin-first scenario, there exists a pair of counterpropagating chiral edge states (with opposite spins) in the gap so that a quantized Hall state at \( \nu = 0 \) appears.4,6 These states provide a dominant contribution to the conductivity, while the bulk transport is suppressed by the energy gap. This leads to divergence of the longitudinal resistivity \( \rho_x \) and smooth zero crossing of the Hall resistivity \( \rho_{yx} \). In the valley-first scenario, no edge states exist in the gap and the divergence of both \( \rho_x \) and \( \rho_{yx} \) at \( \nu = 0 \) is expected.3,5

Since an unconventional quantum Hall state at \( \nu = 0 \) does not rely on relativistic dispersion of excitation, which is a specific case of graphene, it can be realized in other materials where two-dimensional (2D) electrons and holes coexist. The wide CdHgTe/HgTe/CdHgTe quantum wells, where separation of the size-quantized subbands is relatively small, are of particular interest in this connection.7 The 2D conduction (c) band in such systems is formed from the first heavy-hole subband (h1) whose effective mass is positive,8 while the second heavy-hole subband (h2) forms the upper part of the 2D valence (v) band. Owing to a uniaxial strain of the HgTe layer, which is caused by the lattice mismatch of HgTe and CdTe9 or CdHgTe,10 the energy spectrum of the h2 subband is essentially nonmonotonic and has maxima away from the \( \Gamma \) point of the 2D Brillouin zone. Depending on the well width, strain strength, and interface orientation, the band structure can be of the two following kinds: indirect-gap 2D semiconductor and 2D semimetal (Fig. 1), which differ, respectively, by the absence or presence of the overlap of h1 and h2 subbands. In both these cases, a variation of the Fermi energy may cause a transition between holelike (\( \sigma_{xy} > 0 \)) and electronlike (\( \sigma_{xy} < 0 \)) behavior, as is understandable even from a classical transport picture. Indeed, multiple intersections of the Fermi level with nonmonotonic energy spectrum (Fig. 1) lead to a complicated Fermi surface for 2D electrons (Fermi arc) composed of more than one closed branch providing both electronlike and holelike orbits in the presence of the perpendicular magnetic field.11

Recently, it has been demonstrated that the Hall conductivity of CdHgTe/HgTe/CdHgTe quantum wells of 20 nm width and [013] interface direction exhibits a quantized plateau \( \sigma_{xy} = 0 \) in the magnetic fields of a few tesla when the gate voltage is varied in the vicinity of the charge neutrality point (CNP).12 The universality of the edge-state transport picture suggests the existence of a pair of counterpropagating edge states in such 2D systems, which is equivalent to QHE near the Dirac point in graphene within a spin-first scenario. Despite this similarity, one should point out several differences. Unlike the case of graphene, the observation of the QHE state with \( \sigma_{xy} = 0 \) in HgTe quantum wells does not require ultrahigh magnetic fields. More importantly, the band structure of HgTe quantum wells is very different from that of graphene. Graphene is a gapless 2D material with symmetric and monotonic energy spectrum of electrons and holes. The \( n = 0 \) Landau level resides precisely at the electron-hole symmetric
point (Dirac point), which allows for a $\sigma_{xy} = 0$ QHE when
the fourfold (spin and valley) degeneracy is lifted. In wide
HgTe quantum wells the energy spectrum of the v-band is
nonmonotonic, and the extrema of the c-band and v-band are
shifted in momentum space with respect to each other. The
simplest consequence of such asymmetry is that zero crossing
of the Hall resistivity takes place away from the CNP, as
shown below. The overlap of the c-band and v-band makes
the transport picture even more complicated. The question
about mechanisms leading to opening of the gap responsible
for the $\sigma_{xy} = 0$ QHE in wide HgTe quantum wells requires
further consideration.

In this paper, we present experimental results on the
Hall resistivity in 20 nm wide HgTe-based quantum wells
with [001] interface orientation containing simultaneously
electrons and holes. The densities of carriers in these wells
are considerably smaller those those for the [013]-, [112]-,
and slightly wider [001]-grown wells examined previously
in our experiments.\(^7,10,12,13\) For this reason, we see quantum
features in transport at smaller magnetic fields. Apart from the
existence of the $\sigma_{xy} = 0$ plateau in the dependence of Hall
conductivity on the gate voltage, we have found an unusual
nonmonotonic dependence of the Hall resistivity $\rho_{xx}$ on the
magnetic field. This dependence essentially differs from the
classical N-shaped Hall resistivity expected for electron-hole
systems.\(^7,10\) A theoretical consideration based on calculation
of the Landau level spectrum for our structure qualitatively
explains the main features of our observations and helps us to
uncover a mechanism of transition to the $\sigma_{xy} = 0$ state in the
systems under investigation.

The paper is organized as follows. In Sec. II we describe
the details of the experiment and give experimental results.
Section III presents a theoretical basis for consideration of
magnetotransport in 2D semimetals, Landau level calculation
for wide HgTe quantum wells, and discussion of the results.
The conclusions are briefly stated in the final section.

II. EXPERIMENT

The Cd$_{0.65}$Hg$_{0.35}$Te/HgTe/Cd$_{0.65}$Hg$_{0.35}$Te quantum wells
with [001] surface orientations and the width of 20 nm were
prepared by molecular beam epitaxy. A detailed description

of the sample structure has been given in Refs. 7, 12, and 13.
The top view of a typical experimental sample is shown in
Fig. 2(a). The sample consists of three 50 $\mu$m wide consecutive segments of different lengths (100, 250,
and 100 $\mu$m), and eight voltage probes. The ohmic contacts
to the 2D layer were formed by the in-burning of indium.
To prepare the gate, a dielectric layer containing 100 nm
SiO$_2$ and 200 nm Si$_3$N$_4$ was first grown on the structure
using the plasmochemical method. Then the TiAu gate was
deposited. The rate of the density variation with gate voltage is
estimated as $\alpha = 1.09 \times 10^{15}$ m$^{-2}$ V$^{-1}$. The magnetotransport
measurements in these structures were performed in the
temperature range 0.8–10 K and in magnetic fields up to 5 T
using a standard four-point circuit with a 3–13 Hz alternating
current of 1–10 nA through the sample, which is sufficiently
low to avoid the overheating effects. Several devices from the
same wafer have been studied.

The longitudinal resistivity $\rho_{xx}$, and corresponding Hall
resistivity $\rho_{yx}$, acquired by varying the gate voltage at a constant
magnetic field $B = 5$ T are shown in Fig. 2(a). Figure 2(b)
shows the longitudinal conductivity $\sigma_{xx}$ and Hall conductivity
$\sigma_{xy}$ calculated from the experimentally measured resistivities
by tensor inversion. Possible admixtures of longitudinal and
Hall resistivities, caused by contact misalignment and inhomogeneities,
were removed, by symmetrizing all measured values
for positive and negative magnetic fields. We may see that
calculating the conductivities from the magnetoresistance peak
at the CNP and a zero crossing of the Hall resistance yields a
very small longitudinal conductivity $\sigma_{xx}$ and a quantized zero
plateau in the Hall conductivity $\sigma_{xy}$ at $v = 0$. This behavior,
observed in the fields above 3.5 T, agrees with our previous study of the quantum Hall effect near CNP in wide HgTe quantum wells in samples with [013] surface orientation.12

Now we turn our attention to magnetoresistance measured as a function of the gate voltage with increasing magnetic field, shown in Fig. 3. For higher voltages, corresponding to electronlike conductivity, all the quantized plateaux are already developed in the field of 1 T. In the field of 0.5 T, however, we see only a short plateau at \( v = -1 \) and a weak indication of \( v = -3 \) plateau. For lower voltages, corresponding to holelike conductivity, we do not see any well-developed plateaux up to 3 T, although above 1.5 T there is periodic flattening of the Hall resistance picture possibly suggesting that the effects of Landau quantization become important also for holes.

Surprisingly, the peak of the resistivity and the smooth zero crossing point of the Hall resistivity, which has been identified previously with CNP, both are shifted to higher positive voltage with increasing \( B \). At weak magnetic fields this shift is linear in \( B \), demonstrating, however, a weak shoulder identified as a quantized plateau \( \rho_{yx} = -h/3e^2 \). At \( B \gtrsim 0.45 \) T we see a short plateau where the Hall resistivity almost reaches the resistance quantum \( \rho_{yx} \simeq 0.92(-h/e^2) \), which is accompanied by a deep minimum in \( \rho_{xx} \) in agreement with conventional quantum Hall behavior in a unipolar (\( n \)-type) system. With further increase in magnetic field (\( B > 0.5 \) T) the absolute value of the Hall resistivity drops down. In the interval \( 2 \, \text{T} < B < 3 \, \text{T} \) there appear new features resembling the plateaux with \( \rho_{yx} \) ranging from 0 to \( h/e^2 \) in the chosen interval of gate voltages. The plateau \( \rho_{yx} = h/e^2 \) corresponds to conventional quantum Hall effect in a hole (\( p \)-type) system, and is accompanied by a minimum in \( \rho_{xx} \). However, these Hall plateaux do not exhibit exact quantization, as is typical for the states not yet fully formed in the magnetic field. Moreover, the value of \( \rho_{xy} \) smoothly changes with the gate voltage, which possibly indicates that the Fermi level does not lie in the region of localized hole states between Landau levels and the contribution of bulk delocalized states to transport is essential. Finally, in the region above \( 3.5 \, \text{T} \) \( \rho_{xy} \) demonstrates a complicated nonmonotonic behavior which strongly depends on the gate voltage, while the resistivity \( \rho_{xx} \) starts to grow up sharply.

The observed features of Hall resistance and longitudinal resistivity are discussed in the next section using both classical and quantum approaches to magnetotransport in 2D semimetals.

FIG. 3. (Color online) Longitudinal \( \rho_{xx} \) (a) and Hall \( \rho_{yx} \) (b) resistivity at \( T = 0.9 \) K as a function of the gate voltage for different magnetic fields \( B \): 0.5 (orange), 1 (magenta), 1.5 (black), 2.0 (red), 2.5 (green), and 3 (blue).

FIG. 4. (Color online) Longitudinal \( \rho_{xx} \) (a) and Hall \( \rho_{yx} \) (b) resistivities at \( T = 0.9 \) K as functions of the magnetic field for different gate voltages near CNP, from 0.81 to 0.6 V with step 0.015 V. The highest and the lowest gate voltages are shown by broader (colored) lines. The red line corresponds to CNP (\( n_e \simeq n_h \)), while the blue line corresponds to \( n_h - n_e \simeq 1.6 \times 10^{10} \, \text{cm}^{-2} \).
We use the following parameters. The conduction- and valence-band extrema of Fig. 5, the components of the conductivity tensor at low temperature are the sums of contributions from two groups of carriers near the extrema of the conduction and valence bands (i.e., the electron and hole contributions):

$$\sigma_{xx} = \sigma_{xx}^{(e)} + \sigma_{xx}^{(h)}$$
$$\sigma_{xy} = \sigma_{xy}^{(e)} + \sigma_{xy}^{(h)}$$

(2) where \( \beta_i = m_i/|e|\tau_i \) are the inverse mobilities of electrons and holes expressed through the corresponding effective masses \( m_i \) and scattering times \( \tau_i \). The signs \( - \) and \( + \) in \( \sigma_{xy}^{(i)} \) stand for electrons and holes, respectively. The expressions (2) and (3) turn out to be analogous to known expressions for two-component electron-hole systems (see for example, Ref. 16).
The introduction of the effective mass \( m_e \) is natural, since the c-band spectrum is isotropic and parabolic in a wide energy range. The effective mass \( m_k \) is introduced as \( m_k = \sqrt{m_1m_2} \), where \( m_1 \) and \( m_2 \) are the masses characterizing the v-band energy spectrum along the main axes of the ellipses. From our calculations (Fig. 5), we find \( m_e = 0.03m_0 \) and \( m_h \approx 5m_e \), so the density of hole states (taking into account fourfold valley degeneracy) is about 20 times greater than the density of electron states. This means that at low hole densities the Fermi energy is pinned near the valence band extremum, and the parabolic approximation for \( \epsilon_g \) in the valence band is justified. The expressions (2) and (3) lead to the Hall resistivity in the form

\[
\rho_{yx} = -\frac{B}{|e|} \left[ \left( n_e - n_h \right)^2 B^2 + \left( n_e \beta_e^2 - n_h \beta_h^2 \right)^2 \right].
\]

(4)

Since \( n_h \gg m \) and, consequently, \( \beta_h \gg \beta_e \), the condition for zero \( \rho_{yx} \) at low \( B \) is realized at \( n_h > n_e \), away from the CNP. The gate voltage \( V_g^{(i)} \) corresponding to zero crossing can be found from Eq. (4) combined with the expression \( n_e - n_h = \alpha(\epsilon_b - V_g^{(i)}) \), where \( \alpha \) is a constant specified for our sample in the previous section. With increasing \( B \), the voltage \( V_g^{(i)} \) moves towards the CNP gate voltage \( V_g^{(CNP)} \), first linear with \( B \), then saturating in the close vicinity of the CNP. Thus the behavior of the zero crossing point shown in Fig. 3(b) follows from the classical expression (4). The same expression describes N-shaped magnetic-field dependence of the Hall resistance at fixed \( V_g \), under the condition that \( n_h > n_e \). However, the Hall resistance plotted in Fig. 4(b) is different from the simple N-shaped dependence and cannot be described within the classical theory.

To account for the quantum features of the magnetotransport in our system, it is important to get an idea of how the energy spectrum is quantized in the magnetic field. To calculate the Landau levels (LLs) of HgTe quantum wells, we have used isotropic approximation in the Kane Hamiltonian, when the Luttinger parameters \( \gamma_2 \) and \( \gamma_3 \) both are replaced by \( (\gamma_2 + \gamma_3)/2 \). In this approximation, the problem is greatly simplified, since the columnar eigenstates of the 6 \( \times \) 6 matrix Kane Hamiltonian become representable in the form (see also Ref. 17 for 4 \( \times \) 4 Luttinger Hamiltonian)

\[
(u^1_n \psi_{n-2}, u^2_n \psi_{n-1}, u^3_n \psi_{n-3}, u^4_n \psi_{n-2}, u^5_n \psi_{n-1}, u^6_n \psi_n)^T,
\]

(5)

with the oscillatory functions \( \psi_m(n \geq 0) \) satisfying the relations \( k_-\psi_m = \psi_{m+1} \sqrt{2n/T} \) and \( k_+\psi_m = \psi_{m-1} \sqrt{2(n+1)/T} \), where \( T = \sqrt{\hbar/|e|B} \) is the magnetic length. The states are, therefore, described by the LL number \( n \), and for each \( n \) one can find numerically the set of the components \( u^m_n(z) \) \((m = 1, 2, \ldots, 6)\) and the corresponding discrete energies \( \epsilon_j \), where the number \( j \) accounts for both subband number and spin state. These energies, for two chosen values of the magnetic field, are plotted in Fig. 6.\(^{18,19}\) We point out the most essential properties of the LL spectrum. There are two states for each subband with LL number \( n \geq 3 \); these states differ by projection of spin. For \( n = 0, 1, \) and \( 2 \) there is only one spin state within each heavy-hole subband, because the third component of the wave function (5) (spin-up heavy-hole component) is missing for these particular LL numbers. The levels originating from the c-band rapidly go up with increasing \( B \), except the first level (denoted as LA), which slowly moves down with the rate approximately 0.25 meV/T. In contrast, the levels originating from the v-band form a dense set, especially near the band extremum, and are slowly shifted with increasing magnetic field. The v-band levels \( n = 1 \) and \( n = 2 \) (the latter is denoted as LB) are an exception, because they move up rather rapidly with increasing field. At \( B \approx 2.8 \) T the level LB becomes the upper one in v-band. The empty circles in the left panel indicate position of the next two c-band Landau levels at \( B = 0.3 \) T. The dashed line in the right panel shows the position of the Fermi energy corresponding to a quantized plateau \( \alpha_{st} = 0 \) near the CNP.

![FIG. 6. Calculated Landau levels for a 20 nm symmetric Cd\(_{0.65}\)Hg\(_{0.35}\)Te/HgTe quantum well for \( B = 1.5 \) T and \( B = 4 \) T. Only two sets of levels originating from two principal 2D subbands are shown. The levels denoted as LA (the first one in c-band) and LB (\( n = 2 \) level in v-band) undergo crossing at about 2.8 T. Starting from \( B \approx 3.5 \) T, the level LB becomes the upper one in v-band. The empty circles in the left panel indicate position of the next two c-band Landau levels at \( B = 0.3 \) T. The dashed line in the right panel shows the position of the Fermi energy corresponding to a quantized plateau \( \alpha_{st} = 0 \) near the CNP.](image-url)
find this discrepancy surprising, because the calculation puts aside many unknown factors (such as deviation of the quantum well potential from a simple rectangular one, partial relaxation of the strain inside the HgTe layer, etc.) which may shift the quantization energies within several meV. In weaker fields, we observe a signature of a plateau at c-band filling factor 3 (ν = −3) but do not observe any plateau at ν = −2. This fact becomes understandable from our numerical calculation showing that two next to level LA c-band LLs are almost degenerate at low B; see empty circles in the left panel of Fig. 6. Since these are the states with different spin numbers, the nature of the degeneracy is described in terms of negligible spin splitting for low-lying c-band states in weak magnetic fields.

The absence of any sizable contribution of holes to transport at CNP means that both longitudinal and Hall conductivities of holes are much smaller than e²/h. Though such small hole conductivities cannot be entirely described within the classical (Drude) theory, it is likely that the holes experience strong localization. With semimetallic overlap of 1.1 meV, the hole density at CNP is estimated as n_h ≈ 1.2 × 10¹⁰ cm⁻². Since the density of states in v-band is large, the presence of disorder makes it possible that the majority of holes, at such small n_h, occupy the tail of the density of states below the mobility threshold.

In the intermediate region (from 0.45 to 3 T), a single c-band level LA is occupied. The electron density n_e increases linearly with the magnetic field. The difference n_h − n_e is fixed by the gate, the hole density n_h increases as well, and the holes become active in transport, causing a positive contribution to the Hall conductivity, and, consequently, a positive deviation of the Hall resistivity from −h/e². The increase of hole density by the gate leads to a similar effect. The components of the conductivity tensor are described by Eq. (2) with constant electronic components σₓₓ(e) = 0 and σᵧᵧ(e) = −e²/h which can be treated also as a contribution of the chiral edge state originating from level LA to transport. Thus the dependence of resistivity on magnetic field and gate voltage is determined by the hole components of the conductivity tensor. The absence of a detailed knowledge of hole conductivity in the intermediate magnetic field region does not allow us to describe the Hall resistivity quantitatively. Whereas the monotonic change of the Hall resistivity in the region below 1.7 T might be explained within a classical representation of σₓₓ(h) and σᵧᵧ(h), the flat (plateau-like) features at higher fields possibly suggest that Landau quantization of holes already starts and makes σₓₓ(h) and σᵧᵧ(h) nearly independent on B at B > 1.7 T. The fact that the value of ρₓᵧ still smoothly changes with the gate voltage everywhere in this region means that the hole LLs are not well resolved; indeed these LLs form a dense set according to our numerical calculations. The contribution of bulk delocalized hole states to transport is, therefore, essential, and increases proportional to the hole density controlled by the gate.

In the high magnetic field region (above 3 T), the behavior of Hall and longitudinal resistances is determined by another important factor, the influence of the v-band level LB, which rapidly goes up with increasing magnetic field. The reduction of the hole contribution to Hall resistivity occurs when this level passes the Fermi level. According to our calculations, level LB crosses the upper one of the other v-band LLs approximately at B = 3.6 T. As this has happened, level LB itself becomes the upper one in the v-band and accumulates the majority of hole density. The remaining fraction of hole density, equal to n_h − n_v, is much smaller than the capacity of a single LL (at B = 4 T the capacity of a single LL is about 10¹¹ cm⁻²). Therefore, the Fermi level resides in the region of localized states in the tail of the next v-band LL, as shown in the lowest panel of Fig. 7, and bulk contribution to conductivity is negligible. The transport is determined by a pair of counterpropagating chiral edge states originating from LLs LA and LB, so the Hall conductivity σₓᵧ is expected to be zero. Since small variations of the gate voltage do not shift the Fermi level from the region of localized states, a plateau σₓᵧ = 0 appears in the gate-voltage dependence of σₓᵧ; see Fig. 2(b). The fact that we observe such a plateau starting from 3.5 T is consistent with the calculated behavior of LLs and emphasizes the crucial role of the level LB in magnetotransport in wide (semimetallic) HgTe wells. The large longitudinal resistance seen in Figs. 2(a) and 4(a) in this region is caused by scattering between counterpropagating chiral edge states, which is enhanced in the magnetic field. Such large resistance is observed also in nonlocal magnetotransport measurements in similar samples, 21 which provides an additional confirmation of the picture of counterpropagating chiral edge modes in high magnetic fields.

A description of the longitudinal and Hall resistivities in the regime of counterpropagating edge states requires a more detailed consideration, 4 because in view of strong scattering between these edge modes the edge-state transport is suppressed and small bulk contribution to conductivity might be important as well. We apply the formalism developed in Ref. 4 to a system comprising two edge states of electron (e) and hole (h) kind and a continuum of bulk states. The scattering between the edge states and the scattering between bulk states and each of the edge states is characterized by mean free path...
Depending on parameters, $\rho_{yx}$ may behave in different ways, and can be positive or negative. Assuming, for example, that both $\sigma_{xy}^{(b)}$ and $\sigma_{xx}^{(b)}$ are exponentially small, one cannot say that their ratio is small as well, so both contributions in the square brackets of Eq. (7) remain essential. If, in addition, $\sigma_{xy}^{(b)}/\sigma_{xx}^{(b)} \ll (\gamma L_y)^{-1}$, one obtains a much simpler result, $\rho_{yx} = \mu h/2 e^2$ with $\mu = (g_h - g_e)/(g_h + g_e)$. This expression shows that $\rho_{yx}$ is smaller than $h/2 e^2$ and its sign is determined by the bulk-edge scattering. In experiment, we see a complicated, variable-sign behavior of $\rho_{yx}$ as function of $B$ in Fig. 4(b) in the region of high magnetic fields. Without knowing dependence of the parameters entering Eq. (7) on magnetic field and gate voltage, we cannot describe this behavior in detail. Nevertheless, when $\sigma_{xy}$ is calculated from the expressions (6) and (7) by tensor inversion, the large value of $\rho_{xx}$ guarantees that $\sigma_{xy} \ll e^2/h$ regardless of the behavior of $\rho_{yx}$. This result correlates with the existence of the zero-conductivity plateau demonstrated in Fig. 2(b).

\[ \rho_{xx} = \rho_{xx}^{(b)} \left[ \frac{\sigma_{xx}^{(b)}}{\sigma_{xx}^{(b)}} + \frac{1}{L_y} \frac{g_h - g_e}{\gamma(g_e + g_h) + g_e g_h} \right], \]

where $L_y$ is the width of the sample, and $\rho_{xx}^{(b)}$ is the bulk resistivity expressed by a standard tensor inversion through the bulk conductivities $\sigma_{xx}^{(b)}$ and $\sigma_{xy}^{(b)}$. Since both these conductivities are much smaller than $e^2/h$ in the regime under consideration, and backscattering is strong, $\gamma L_y \gg 1$, the resistivity $\rho_{xx}$ becomes much larger than the resistance quantum, as seen in Figs. 4(a) and 2(a). The Hall resistance is

\[ \rho_{yx} = \rho_{xx}^{(b)} \left[ \frac{\sigma_{xy}^{(b)}}{\sigma_{xx}^{(b)}} + \frac{1}{L_y} \frac{g_h - g_e}{\gamma(g_e + g_h) + g_e g_h} \right]. \]
18 Of course, the numbering of the LLs can be chosen in a different way within each subband and each spin subsystem. The numbering used in this paper is based solely on the fact that \( n \) is the number of the oscillatory function in Eq. (5).