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NTRTP: A new reduction to the pole method at low latitudes via a nonlinear thresholding

Henglei Zhang\textsuperscript{a,b}, Yára R. Marangoni\textsuperscript{a,*}, Xiangyun Hu\textsuperscript{b}, Renguang Zuo\textsuperscript{c}

\textsuperscript{a} Geophysics Department, Instituto de Astronomia, University of Sao Paulo, Sao Paulo, SP, Brazil
\textsuperscript{b} Institute of Geophysics and Geomatics, China University of Geosciences, Wuhan 430074, China
\textsuperscript{c} State Key Laboratory of Geological Processes and Mineral resources, China University of Geosciences, Wuhan 430074, China

**A B S T R A C T**

We present a stable reduction to the pole (RTP) approach using a nonlinear threshold method with better RTP performance for magnetic data at low latitudes. In the new nonlinear thresholding RTP (NTRTP) method, the routine RTP operator is divided into two parts (the real part and imaginary part), which are modified respectively based on a nonlinear threshold to suppress the large amplitude linked to instability. It is tested on a couple of synthetic data (one is modeled at magnetic equator) and a field case from central Brazil (with a low inclination of $-5^\circ$) and compared with two existing RTP methods. The proposed method in this study performs stable and estimates more accurate amplitudes for RTP field than the existing methods used in this study.

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1. Introduction

Reduction to the pole (RTP) transforms an observed magnetic anomaly into an anomaly that would be measured at the north magnetic pole. This relocates asymmetrical magnetic anomalies to be over their sources, thus making magnetic interpretation easier. However, magnetic RTP at low latitudes (RTP-L), especially at equator (RTP-E), routinely computed in the wavenumber domain, is notoriously unstable. Besides, when RTP is applied on determining the total magnetization direction, the calculation becomes unstable when the inclination of the inducing field and/or the total magnetization field is close to zero the absolute value is smaller than $5^\circ$ and results in an incorrect estimated magnetization direction (Gerovska et al., 2009).

The effect of RTP-L instability is from all the wavenumber in two wedge-shaped segments, and is hard to stabilize. In the downward continuation (DC), the instability is from the large amplitudes of the DC operator in high wavenumber, so it could be stabilized by tapering the high wavenumber anomalies (Pašteka et al., 2012; Zhang et al., 2013).

In application, a lot of research prefer the use of the analytic signal amplitude (ASA) to identify magnetic anomalies instead of RTP (Ansari and Alamdar, 2009; Keating and Sailhac, 2004). To overcome the instability of RTP-L, the existing works have focused upon constructing an approximate RTP operator with suppressing the routine RTP operator in and near the declination direction. Grant and Dodds proposed the pseudo-inclination approach (see MacLeod et al., 1993). Mendonça and Silva (1993) presented a formulation of the RTP operator based on the truncated Taylor’s series of the theoretical expression for the RTP operator in the wavenumber domain (TTRTP). To attenuate the undesirable high wavenumber noise due to shallow and small sources, the upward continuation is also applied in this research. Hansen and Pawlowski (1989) proposed the wiener filtering method based on WFRTP using the energy balance technique to estimate the noise-to-signal power ratio. To reproduce the observed data, Keating and Zerbo (1996) emphasize the need of better reproduction of the data as a factor for improved RTP results. They computed the difference between the observed data and the projected result, reduced the difference to the pole, and added this secondary RTP result to the primary one. This process may be repeated more than once and may help recover amplitude information that was filtered out in the first run. Phillips (1997) presented the azimuthal filtering method to taper to routine RTP operator in the unstable area determined by a threshold angle $\beta_0$. Guo et al. (2013) proposed the antisymmetric factor approach for stable RTP with a modified function instead of the routine RTP factor in the unstable area determined by a threshold angle $\beta_0$.

Besides, some research focused on the inversion idea to stabilize the RTP-L. Li and Oldenburg (2001) discussed an inverse formulation for the RTP operation, that the RTP field is constructed by solving an inverse problem in which a global objective function is minimized subject to fitting the observed magnetic field. A method called as the equivalent...
source approach is also discussed, e.g., Li and Oldenburg (2000), Guspí and Novara (2009), Grandis (2013), and elsewhere.

Li (2008) analyzed the RTP-L methods and stated that determining an optimal parameter for the RTP-L techniques is important, because the RTP results are sensitive to the parameter. So he suggested using some different geologic and geophysical checks to select a reasonable RTP result.

In this work, we first present a new approach, called the nonlinear thresholding RTP method (NTRTP), which works at low magnetic latitudes, especially at the magnetic equator. To determine an optimal parameter for the RTP-L methods, the correction method between the RTP fields and the canonical invariant of the magnetic gradient tensor ($I_1$) is discussed and applied on the model tests and the field data for the proposed method NTRTP.

2. The method

If there is no remnant magnetization or the direction of remnant magnetization is in the direction of the main magnetic field, the RTP operator is

$$Q = \frac{(\sqrt{u^2 + v^2})^2}{(i \cdot (u \cdot L_0 + v \cdot M_0) + N_0 \cdot \sqrt{u^2 + v^2})^2},$$

where $L_0 = \cos I \cdot \cos D, M_0 = \cos I \cdot \sin D, N_0 = \sin I, I$ and $D$ are the wavenumbers in the $x$- and $y$-direction. For RTP at low latitude, when the points locate on or near $u = - \tan D \cdot v$, as shown in Fig. 1, the operator will produce very large amplitudes which lead to instability.

Fig. 1. The character of RTP operator in the wavenumber domain. The filled area will involve in large values at low latitudes leading to be unstable.

Fig. 2. The amplitude of the RTP operator corresponding to the real part (a) and the imaginary part (b). The magnetic inclination is 20° and the declination is 0° for the routine RTP operator. For the AFRTP operator, the threshold angle is 40° and the exponential power is 1. For the TTRTP operator, the parameter $M$ is set as 5.

Fig. 3. The synthetic model data set. The model locations are indicated by the black lines. (a) The magnetic response with an inclination of 0° and a declination of 0°. (b) The magnetic response with an inclination of 90° and a declination of 0°.
The truncated Taylor series approximation of the RTP method (TTRTP) proposed by Mendonça and Silva (1993) is defined as

$$F_{z\perp}(u, v) = F_t(u, v) \times \sin(I) - \frac{i}{C_1} \cos(I) - \frac{D}{C_1} \cos(\theta - D)$$

where $F_{z\perp}(u, v)$ and $F_t(u, v)$ denote the Fourier transform of RTP anomaly and observed anomaly, respectively. $\theta$ is the spectral azimuth related to $u$ and $v$. $H$ is the continuation distance with respect to the observation plane, $K = 2\pi \sqrt{u^2 + v^2}$. The parameters of the Taylor series expansion term $M$ and $H$ are defined by

$$M = \frac{\ln(1 - \sqrt{a})}{2 \cdot \ln(\cos(I) \cdot \cos(\Delta \theta))} - 1, \quad e^{-i k_i H} = \text{cof},$$

where $a$ is a positive number between zero and one and $\Delta \theta$ is a small angle different from zero. $k_i$ is the wavenumber around which most of the signal power is concentrated, and $k_s$ is the smallest wavenumber beyond which the noise starts to dominate the spectrum. $\text{cof}$ is the specified desired ratio. Because the parameters $a$ and $\Delta \theta$ are not known, it is also difficult to determine the Taylor series expansion term, $M$.

The azimuthal filtering for RTP (AFRTP) designed by Phillips (1997) and analyzed by Li (2008) tapers the RTP operator within $\pm \beta$ relative to the direction of the declination by the following factor

$$\text{filter} = (\sin(\frac{\pi \cdot |\theta - D + 90|}{2 \cdot \beta}))^p.$$  

The exponential power parameter $p$ determines the falloff rate of the taper and often has a relatively weak effect on the RTP result. In our model and case studies, we use the same value ($p = 4$) as discussed in Li (2008).

### 3. The proposed method

For the RTP operator shown in Eq. 1, we consider it as

$$Q = \text{Re} + i \cdot \text{Im},$$

where $\text{Re}$ and $\text{Im}$ denote the real part and imaginary part of the routine RTP operator.
The basic reason of instability for RTP at low latitudes is the large amplitudes of Re or Im located inside the unstable areas. In our study, we suggest using a special filter to stabilize the RTP at low latitudes where $x$ means the values of Re or Im, $A$ means the amplitude threshold, and $F$ means a special filter, which we will determine later.

Then, the proposed RTP operator in this study, called as the nonlinear thresholding for magnetic RTP at low latitudes (NTRTP), is defined as

$$Q_{\text{new}} = \text{Re} \cdot F_1 + i \cdot \text{Im} \cdot F_1.$$  \hspace{1cm} (7)

For the filter $F$, it could be determined by many types of functions with the amplitudes damping from 1 to 0. Here we use a sine function to replace the original operator where the value is bigger than the determined threshold. The format of the function is as follows

$$x_{\text{new}} = \begin{cases} x, & \text{abs}(x) \leq 0.9A \\ \text{sign}(x) \cdot \left( 0.9 \cdot A + 0.1 \cdot A \cdot \sin \left( \frac{\theta_2 - \theta_1}{\theta_2 - \theta_1} \pi \right) \right), & \text{abs}(x) > 0.9A \\ \end{cases} \hspace{1cm} \text{abs}(x) \leq 0.9A.$$  \hspace{1cm} (8)

where $x$ means the values of Re or Im, $\theta_1$, and $\theta_2$ denote the two edges of the wedge-shaped segment where its amplitudes are larger than 0.9 $A$. Eq. 8 shows that the smaller the amplitude threshold $A$, the smoother the RTP result.

As shown in Fig. 2, after stabilization based on Eq. 8, the routine RTP operator with the amplitudes of Re or Im larger than the threshold will be changed to the red lines, while the other places smaller than the threshold will keep the same. It means that the NTRTP operator keeps the same as the routine RTP operator outside of the unstable area. While for both TTRTP (the green line in Fig. 2) and AFRTP (the blue line in Fig. 2) methods, a larger area of the routine RTP operator is modified for stabilizing.

A-priori parameter needed for NTRTP is the threshold $A$, which will be optimal determined by the correlation method. The correlation method belongs to the group of research for determination of the inclination and declination based on finding the maximum of the correlation coefficient between RTP results and the analytic signal (Dannemiller and Li, 2006; Gerovska et al., 2009). In our previous study (Zhang et al., 2014), we show that the canonical invariant of the magnetic gradient tensor ($I_1$) reduces the magnetization direction better than the ASA field. So we perform the correlation coefficient between the $I_1$ and the computed RTP fields, and the optimal threshold will produce the maximum correlation coefficient.
4. Test case of model data

We use a couple of model tests to compare the proposed method NTRTP with the existing methods such as the AFRTP method (Li, 2008; Phillips, 1997), and the TTRTP method (Mendonça and Silva, 1993). Fig. 3a shows the magnetic response from three vertical-sided prisms whose tops are at depths of 2, 2.5, and 3 km for prisms A, B, and C, respectively; and bottoms are at depths of 42, 42.5, and 43 km for prisms A, B, and C, respectively. The edge locations are indicated by the black lines. The prisms have magnetization contrasts of 8, 5, and 10 A/m. The inducing field has an inclination of 0° and a declination of 0°. The computed model anomaly at 0.32 km spacing is shown in Fig. 3a (corrupted with additive Gaussian random noise with zero mean and standard deviation of 1% of the maximum amplitude of the theoretical anomalies). Fig. 3b shows the magnetic response with an inclination of 90° and a declination of 0° for the same model. A perfect RTP field should correspond to Fig. 3b.

First, the analytic signal amplitude (ASA) is applied on Fig. 3a and b. As shown in Fig. 4, the ASA on the vertical direction magnetization (Fig. 3b) produces correct source’s locations, while the ASA on Fig. 3a gives incorrect result. It means that the ASA cannot solve the non-vertical magnetization effect, and the RTP is necessary.

![Fig. 8](image)

**Fig. 8.** The optimal determined parameters for the model data shown in Fig. 3a from the AFRTP, TTRTP, and NTRTP methods. The black lines show the correlation coefficient between the computed RTP results and the real RTP field shown in Fig. 3b, and thus, the optimal parameter produces the maximum correlation coefficient. The dotted line in Fig. 8c shows the correlation coefficient between the computed RTP results and the canonical invariant of the magnetic gradient tensor, which is used for determining the threshold for the NTRTP method.

![Fig. 9](image)

**Fig. 9.** (a) The magnetic response with an inclination of 8° and a declination of −50° for the induced field, and an inclination of 8° and a declination of 50° for the total magnetization field. The model locations are indicated by the black lines. The RTP fields from the AFRTP, the TTRTP, and the NTRTP are shown in Fig. 9b, c, and d, respectively. Notice that Fig. 9b–d are plotted with the same color scale as Fig. 3b.
Figs. 5–7 show the RTP results by AFRT, TTRT, and NTRT methods, respectively. A fair comparison of the mentioned methods is based on an optimum parameter used for each method. As shown in Fig. 8, the optimal values are $\beta = 20$, $M = 27$, and $A = 9$ for AFRT, TTRT, and NTRT methods, respectively. We note that the dotted line shown in Fig. 8c denotes the correction coefficients between the RTP fields and the canonical invariant of the magnetic gradient tensor, and determines $A = 11$ for the NTRT method which is close to the optimal one.

Comparison of Figs. 5a, 6a and 7a shows that the main anomalies of the RTP fields are over the sources, but the amplitudes (minimum and maximum values) of NTRT field are more similar to Fig. 3b (note that the Figs. 3b, 5a, 6a and 7a are plotted with the same color scale). Figs. 5b, 6b and 7b show the ASA fields based on the corresponding RTP results shown in Figs. 5a, 6a and 7a, respectively. All of the three ASA results from the properly computed RTP fields show the sources' locations better than the ASA result shown in Fig. 4a from the magnetic field at magnetic equator. On the other hand, the ASA shown in Fig. 7b based on the NTRT field shows less elongated formats towards the declination direction than both Figs. 5b and 6b.

Fig. 9a shows the magnetic response from the same model shown in Fig. 3 with an inclination of $8^\circ$ and a declination of $-50^\circ$ for the induced field, and an inclination of $8^\circ$ and a declination of $50^\circ$ for the total magnetization field. The field shown in Fig. 9a is corrupted with additive Gaussian random noise with zero mean and standard deviation of 10% of the maximum amplitude of the theoretical anomalies. Fig. 9b–d shows the RTP results by AFRT, TTRT, and NTRT methods, respectively. A fair comparison of the mentioned methods is based on an optimum parameter used for each method. As shown in Fig. 10, the optimal values are $\beta = 5$, $M = 130$, and $A = 5$ for AFRT, TTRT, and NTRT methods, respectively. We note that the dotted line shown in Fig. 10c denotes the correction coefficients between the RTP fields and the canonical invariant of the magnetic gradient tensor, and determines $A = 5$ for the NTRT method, which is the same as the optimal one. In this case all of the three RTP fields shown in Figs. 5b–d are more closed to the real RTP field shown in Fig. 3b than the RTP fields shown in Figs. 5a, 6a, and 7a computed from the magnetic field at equator shown in Fig. 3a. Comparison of Fig. 9b–d shows that the RTP field from the NTRT method is more similar to Fig. 3b (note that the Figs. 3b and 9b–d are plotted with the same color scale).

Since the RTP factor at low latitudes is sensitive to the noise, the RTP fields discussed above from the noisy data shown in Figs. 3a and 9a are based on tapering the noise in high wavenumber. As shown in the radially averaged power spectrum (Fig. 11), we keep 26% and 23% of low end wavenumbers for the noisy data shown in Figs. 3a and 9a, respectively.

We use correlation coefficient (Corr) and root mean square (RMS) as measures to compare the RTP results of the AFRT, TTRT, and NTRT methods. As shown in Table 1, the NTRT method obtains the highest Corr and lowest RMS with respect to the theoretical value.

### 5. Application to a real case

We apply the RTP techniques discussed above to a geological structure survey in the north of Goiás state of Central Brazil. Fig. 12a shows the magnetic anomalies with an inclination of $-5^\circ$ and a declination of $-15^\circ$.

Because of the low magnetic latitude, most of the magnetic values are negative, they are hard for interpretation: the relationship between the magnetic body and the magnetic anomaly is not clearly, i.e., there are few anomalies corresponding to the Volcano Sedimentary Sequence of Palmeirópolis (VSSP) and the Mafic Ultramafic Complex of Cana Brava (MUCCB), which areas are shown by black lines. The VSSP is a sequence with amphibolites, schists and metal sediments, from bottom to top. The amphibolite, the main sequence in the area, displays the magnetic anomaly (Carminatti et al., 2003). The MUCCB is a stratified

### Table 1

| Correlation coefficient (Corr) and root mean square (RMS) for models 1 and 2. |
|-------------------------------|-----------------|-----------------|
| **RTP fields** | **Corr** | **RMS (nT)** |
| AFRT, Fig. 5a | 0.85 | 581 |
| TTRT, Fig. 6a | 0.85 | 360 |
| NTRT, Fig. 7a | 0.97 | 139 |
| AFRT, Fig. 9b | 1.00 | 291 |
| TTRT, Fig. 9c | 1.00 | 398 |
| NTRT, Fig. 9d | 1.00 | 120 |
intrusion from a basaltic (olivine–toleitic) magma source with clear fractional crystallization (Carminatti et al., 2003). The intrusions are surrounded by granitic rocks of Pre Cambrian age. In the west part of the map of Fig. 12a we found rocks from the Goiás Magmatic Arc, a feature related with the closure of an ocean in the early Proterozoic. To process the magnetic data better, we try to apply the RTP methods, as discussed in detail below.

Because the observed field is noisy, we keep 32% of low end wavenumbers for the noisy data shown in Fig. 12a based on the radially averaged power spectrum (this figure is not shown). Results from the NTRTP, AFRTP, and the TTRTP are shown in Figs. 12b, 13a, and b, respectively. The parameters of these methods are determined based on Fig. 14, where we set $\beta = 48$, $M = 2$, and $A = 1.2$ for AFRTP, TTRTP, and NTRTP, respectively. Comparison of the results shows that the amplitudes of the AFRTP and the TTRTP methods (Fig. 13) are significantly lower than the NTRTP method (Fig. 12b), and hence cannot be meaningfully plotted with the same color scale. All of the RTP fields improve the resolution of the likely locations of the VSSP and the MUCCB, and maybe some other geological structures, as the Goiás Magmatic Arc (Pimentel et al., 2000) in the west border of the area. These methods reveal a clearer relationship between geological structures and the RTP anomalies. In addition, we note that the NTRTP method is able to display features better than the other two methods.

6. Discussions and conclusions

Similar to the stabilizing downward continued field that suppresses the geological signal at high wavenumber, all of the RTP-L methods

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**Fig. 12.** (a) The magnetic data of a portion of central Brazil with an inclination of $-5^\circ$ and a declination of $-15^\circ$. The grid interval is 500 m and the total number of the actual data points used in the grid is 99,678. (b) The RTP result via the proposed method, NTRTP, performed on the anomalies shown in Fig. 12a. The optimum threshold used in this case is 1.0 (determined based on Fig. 14). The black lines denote the geological bodies’ area: 1 — the Volcano Sedimentary Sequence of Palmeirópolis (VSSP); 2 — the Mafic Ultramafic Complex of Cana Brava (MUCCB).

**Fig. 13.** The RTP results from the AFRTP method (a) and the TTRTP method (b). The optimum values of the threshold angle ($\beta$) for AFRTP and the Taylor series expansion term ($M$) for TTRTP are $65^\circ$ and 1, respectively, determined based on Fig. 14.
based on modification of the routine RTP operator in wavenumber domain will lose the information along the declination, and thus, the obtained result is just an approximate RTP field. Although the magnetic reduction-to-the-equator (RTE) has been proposed to process the magnetic anomaly at low latitudes instead of RTP, we have shown that a properly computed RTP field shows better interpretation than the RTE field. Besides, the equivalent source RTP (ESRTP) could obtain an absolutely stable RTP field at any low latitudes. However, without an efficient inversion, the ESRTP method’s requirements of time and/or computer memory will be substantially greater than the normal RTP methods, rendering the ESRTP method impractical for huge data sets on desktop computers commonly available today.

We have developed a new stable RTP-L method for magnetic data using a nonlinear threshold. The proposed method (NTRTP) does not stabilize the RTP operator as a whole factor as the existing methods done. The NTRTP method divides the routine RTP operator into two parts (the real part and imaginary part), and stabilizes the two parts respectively based on a nonlinear thresholding. It has been demonstrated in the model tests that NTRTP method yields more accurate RTP results and the canonical invariant of the magnetic gradient tensor. The determined parameter for each method is corresponding to the maximum correlation coefficient.

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Fig. 14. The determined parameters for the actual data shown in Fig. 12a from the AFRTT, TTRT, and NTRTP methods. The black lines show the correlation coefficient between the computed RTP results and the canonical invariant of the magnetic gradient tensor. The determined parameter for each method is corresponding to the maximum correlation coefficient.